

Lecture 2

acceleration can be written in 3 ways

1- $a = f(t) = \frac{dv}{dt}$

$$\int dv = \int f(t) dt \Rightarrow v = g(t)$$

2- $a = f(x) = \frac{v dv}{dx}$

$$\int v dv = \int f(x) dx \Rightarrow \frac{1}{2} v^2 = g(x)$$

3- $a = f(v)$

a) $a = \frac{dv}{dt} = f(v)$, $\int \frac{dv}{f(v)} = \int dt$

b) $a = \frac{v dv}{dx} = f(x)$, $\int \frac{v dv}{f(v)} = \int dx$

Ex 11.2 Pg 810

a) $\frac{dv}{dt} = a = -9.81$ (upwards is +ve \therefore downw. acc. is negative)

$$\int_{10}^v dv = -9.81 \int_0^t dt$$

Final value
initial value

$$v \Big|_{10}^v = -9.81 t \Big|_0^t$$

$$v - 10 = -9.81 t$$

$$v = 10 - 9.81 t$$

$$\frac{dy}{dt} = v = 10 - 9.81 t$$

$$\int_{20}^y dy = \int_0^t (10 - 9.81 t) dt$$

$$y - 20 = 10t - \frac{9.81 t^2}{2}$$

$$y = -4.95 t^2 + 10t + 20$$

b) y_{\max} at $v = 0$, $0 = 10 - 9.81 t$ $t = 1.02 \text{ s}$

$$y = 25.1 \text{ m}$$

c) time when ball hits ground $\Rightarrow y = 0$

$$-4.95 t^2 + 10t + 20 = 0$$

$$t = 3.26 \text{ s}$$

Ex 11.3

a) $a = -kv = \frac{dv}{dt}$

$$\int_{v_0}^v \frac{dv}{v} = - \int_0^t k dt$$

v_0 : initial velocity

$$\ln v \Big|_{v_0}^v = -kt$$

$$\ln v - \ln v_0 = -kt, \quad \ln \frac{v}{v_0} = -kt$$

$$\frac{v}{v_0} = e^{-kt}$$

$$v = v_0 e^{-kt}$$



$$b) V = \frac{dx}{dt} = V_0 e^{-kt} \quad \int_0^x dx = V_0 \int_0^t e^{-kt} dt$$

$$x = V_0 \left. \frac{e^{-kt}}{-k} \right|_0^t = \frac{V_0}{-k} (e^{-kt} - 1) \quad x = \frac{V_0}{-k} (e^{-kt} - 1)$$

$$c) a = -kV = \frac{V dv}{dx} \quad \int_{V_0}^V dv = -k \int_0^x dx$$

$$V - V_0 = -kx, \quad V = V_0 - kx$$

~~Integration Rules~~

